

The Real Options Approach to Valuation: Challenges and Opportunities

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FinanceUC Conference, January 2013

My Research on Real Options Valuation

- Mines and Oil Deposits
- Stochastic Behavior of Commodity Prices
- Forestry Resources
- Expropriation Risk in Natural Resources
- Research and Development
- Internet Companies
- Information Technology

Outline of Talk

- Basic ideas about Real Options Valuation
- Solution procedures
- Natural resource investments and the stochastic behavior of commodity prices
- Research and Development Investments

Basic Ideas about Real Options

What are Real Options?

- The Real Options approach is an extension of financial options theory to options on real (non financial) assets
- Options are contingent decisions
 - Give the opportunity to make a decision after you see how events unfold
- Uncertainty and the firm's ability to respond to it (flexibility) are the *source of value* of an option
- Real Option valuations are aligned with financial market valuations

Investment Projects as Options

1. Option to expand a project:

Invest in a negative NPV project which gives the *option* to develop a new project.

2. Option to postpone investment:

Project may have a positive NPV now, but it might not be optimal to exercise the *option* to invest now, but wait until we have more information in the future (valuation of mines).

Investment Projects as Options

3. Option to abandon:

Projects are analyzed with a fixed life, but we always have the *option* to abandon it if we are loosing money.

4. Option to temporarily suspend production: open and close facility.

Traditional Valuation Tools (DCF)

- Require forecasts
 - A single expected value of future cash flows is generally used
 - Difficulty for finding an appropriate discount rate when options (e.g., exit option) are present
- Future decisions are fixed at the outset
 - no flexibility for taking decisions during the course of the investment project

Risk Neutral Valuation

- Lets first look at one aspect of the new approach
- Traditional vs. Certainty Equivalent approach to valuation
- Options a little later

Traditional Approach: NPV

- Risk adjusted discount rate

$$NPV = C_0 + \sum_{t=1}^N \frac{C_t}{(1+k)^t}$$

C_t expected cash flow
in period t

k risk adjusted discount rate

Certainty Equivalent Approach

$$NPV = C_0 + \sum_{t=1}^N \frac{CEQ_t}{(1 + r_f)^t}$$

CEQ_t

certain cash flow that
would be exchanged for risky cash flow
(market based)

Simple Example

- Consider a simplified valuation of a Mine or Oil Deposit
- The main uncertainty is in the commodity price and futures markets for the commodity exist (copper, gold, oil)
- Brennan and Schwartz (1985)
- For the moment neglect options

Traditional vs. CE Valuation

Traditional Valuation:

$$NPV = C_0 + \sum_{t=1}^N \frac{Rev_t - Cost_t}{(1+k)^t} = C_0 + \sum_{t=1}^N \frac{q_t S_t - Cost_t}{(1+k)^t}$$

Certainty Equivalent Valuation:

$$NPV = C_0 + \sum_{t=1}^N \frac{q_t F_t - Cost_t}{(1+r_f)^t}$$

These Results are General

Cox and Ross (1976), Harrison and Kreps (1979) and Harrison and Pliska (1981) show that the *absence of arbitrage* imply the existence of a probability distribution such that securities are priced at their discounted (at the risk free rate) expected cash flows under this *risk neutral or risk adjusted* probabilities (*Equivalent Martingale Measure*).

Adjustment for risk is in the probability distribution of cash flows instead of the discount rate (Certainty Equivalent Approach).

- If markets are complete (all risks can be hedged) these probabilities are unique.
- If markets are not complete they are not necessarily unique (any of them will determine the same market value).
- Futures prices are expected future spot prices under this risk neutral distribution.
- This applies also when r is stochastic.

$$V_0 = E^Q \left[e^{\int_0^T -r_f(t) dt} X_T \right]$$

Using the Risk Neutral Framework to value projects allows us to

- Use all the information contained in futures prices when these prices exist
- Take into account all the flexibilities (options) the projects might have
- Use the powerful analytical tools developed in contingent claims analysis

Real Options Valuation (1)

- Risk neutral distribution is known
 - Black Scholes world (Geometric Brownian Motion)
 - Gold mine is, perhaps, the only pure example of this world

$$F_{0,T} = S_0 (1+r_f)^T$$

Real Options Valuation (2)

- Risk neutral distribution can be obtained from futures prices or other traded assets
 - Copper mine, oil deposits
- More about this later on

Real Options Valuation (3)

- Need an equilibrium model (CAPM) to obtain risk neutral distribution because there are no futures prices
 - R&D projects
 - Internet companies
 - Information technology

Summary: Real Options Valuation

- For many projects, flexibility can be an important component of value
- The option pricing framework gives us a powerful tool to analyze those flexibilities
- The real options approach to valuation is being applied in practice
- The approach is being extended to take into account competitive interactions (impact of competition on exercise strategies)

Solution Procedures

Solution Methods (1)

- Dynamic Programming approach
 - lays out possible future outcomes and folds back the value of the optimal future strategy
 - binomial method
 - widely used of pricing simple options
 - good for pricing American type options
 - not so good when there are many state variables or there are path dependencies
 - Need to use the risk neutral distribution

Solution Methods (2)

- Partial differential equation (PDE)
 - has closed form solution in very few cases
 - BS equation for European calls
 - generally solved by numerical methods
 - very flexible
 - good for American options
 - for path dependencies need to add variables
 - not good for problems with more than three factors
 - technically more sophisticated (need to approximate boundary conditions)

Solution Methods (3)

- Simulation approach
 - averages the value of the optimal strategy at the decision date for thousands of possible outcomes
 - very powerful approach
 - easily applied to multi-factor models
 - directly applicable to path dependent problems
 - can be used with general stochastic processes
 - intuitive, transparent, flexible and easily implemented
 - But it is forward looking, whereas optimal exercise of American options has features of dynamic programming

Valuing American Options by Simulation: A simple least-squares approach

- Longstaff and Schwartz, 2001
- An American option gives the holder the right to exercise at multiple points in time (finite number).
- At each exercise point, the holder optimally compares the immediate exercise value with the value of continuation.
- Standard theory implies that the value of continuation can be expressed as the conditional expected value of discounted future cash flows.

Valuing American Options by Simulation: A simple least-squares approach

- Main idea is that the conditional expected value of continuation can be estimated from the cross-sectional information from the simulation by LS.
- We estimate the conditional expectation function by regressing discounted ex post cash flows from continuing on functions of the current (or past) values of the state variables.
- The fitted value from this cross-sectional regression is an efficient estimator of the conditional expectation function. It allows us to accurately estimate the optimal stopping rule for the option, and hence its current value.

Natural Resource Investments

Valuing Commodity Assets

- First paper on commodities 1982: “The Pricing of Commodity-Linked Bonds”, bonds in which the payout (coupon and/or principal) is linked to the price of a commodity (oil, copper, gold)
- In 1985, “Evaluating Natural Resource Investments” (with M. Brennan), mine and oil deposits could be interpreted and valued as complex options on the underlying commodities. One of the first papers on Real Options.

Stochastic Process for Commodities

- Assumed stochastic process for commodity prices (similar to stock prices) : simplistic assumption. OK for gold but not for other commodities.
- Assumption not satisfactory because supply and demand adjustments induce *mean reversion in commodity prices*

Commodity Prices

- In the next 30 years I wrote (alone and with coauthors) many articles trying to make more realistic assumptions about the process followed by commodity prices. Including about electricity prices where seasonality is important
- Presidential address to the AFA (1997) on “The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging”

Three Factor Model: Actual (Cortazar and Schwartz (2003))

$$dS = (v - y)Sdt + \sigma_1 Sdz_1$$

$$dy = -\kappa ydt + \sigma_2 dz_2$$

$$dv = a(\bar{v} - v)dt + \sigma_3 dz_3$$

Three Factor Model: Risk Neutral

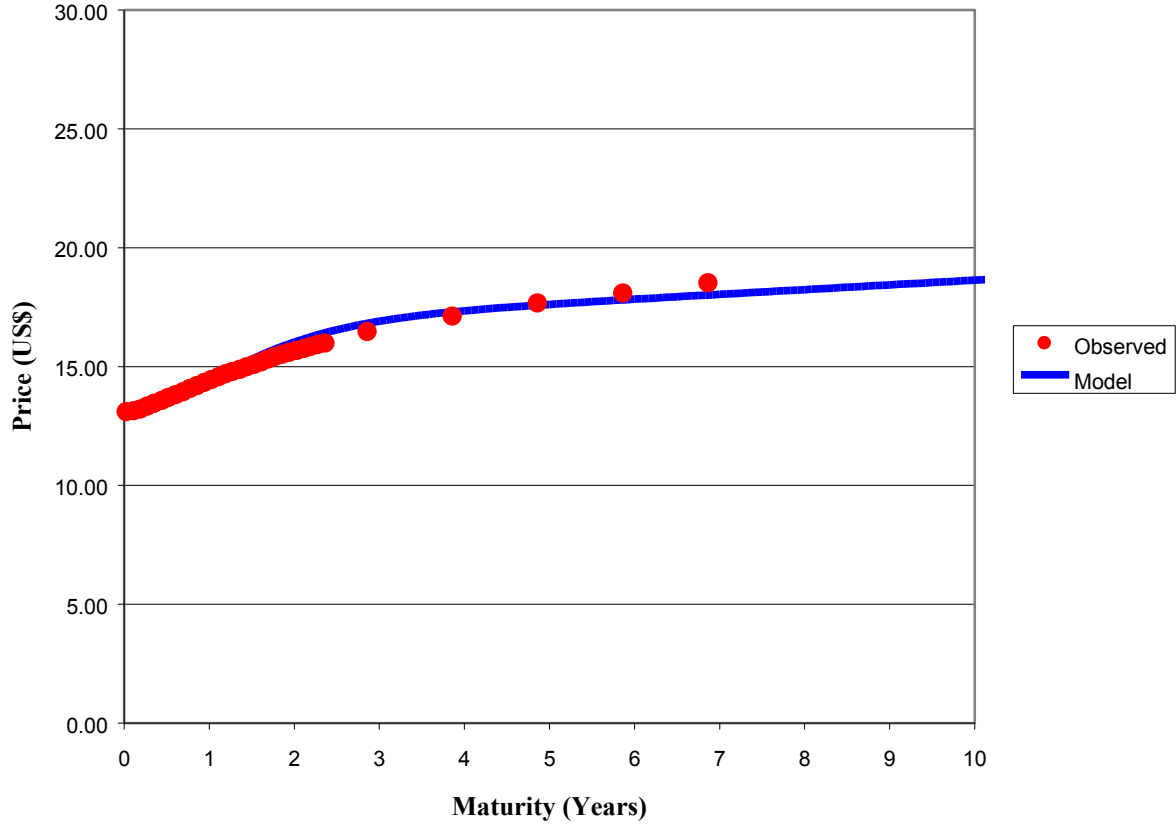
$$dS = (\nu - y - \lambda_1)Sdt + \sigma_1 S dz_1^*$$

$$dy = (-\kappa y - \lambda_2)dt + \sigma_2 dz_2^*$$

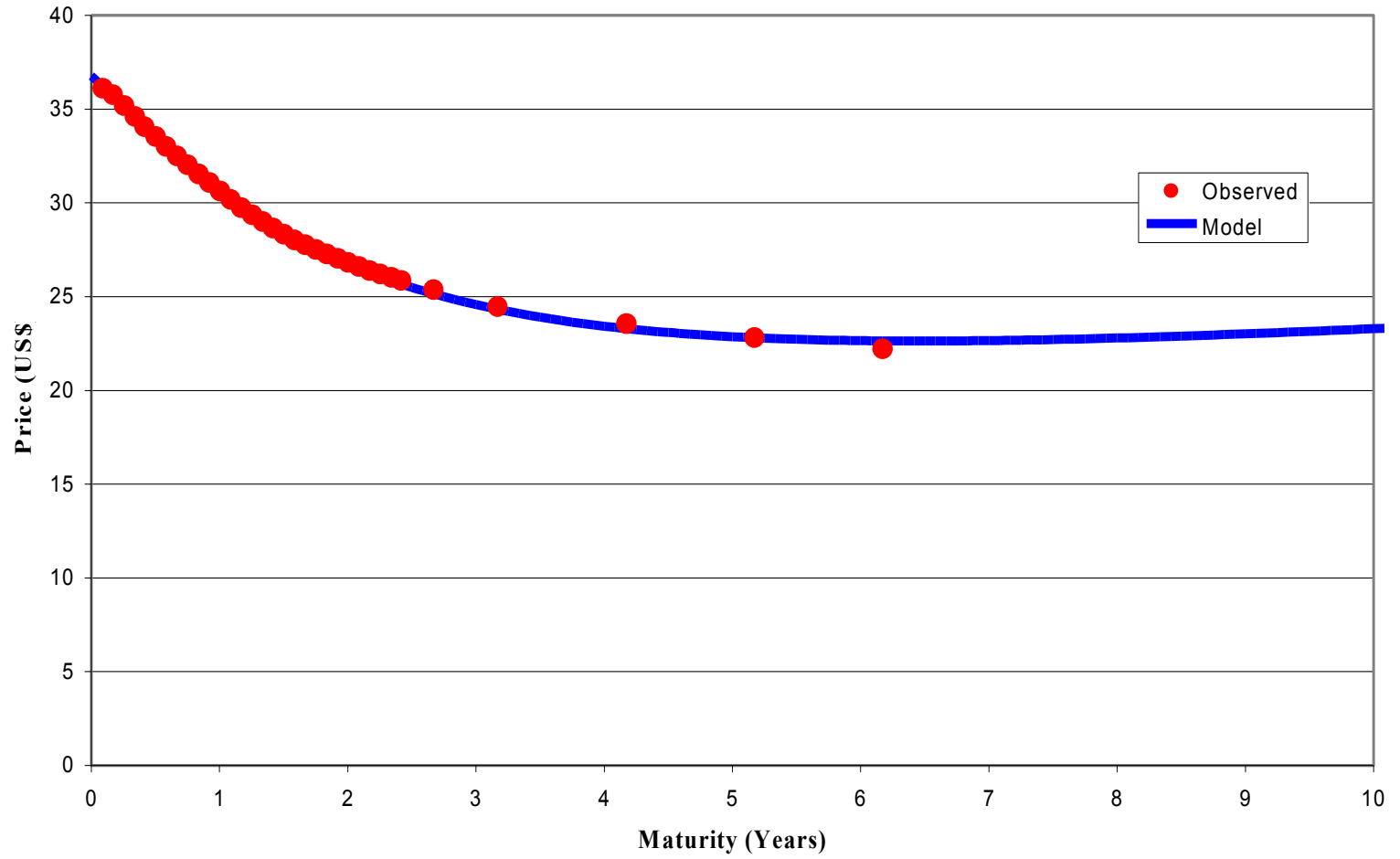
$$d\nu = (a(\bar{\nu} - \nu) - \lambda_3)dt + \sigma_3 dz_3^*$$

We need to make assumptions about the functional form of the market prices of risks

Oil Futures 01/08/99
Three-Factor Model



Oil Futures 10/12/00 Three-Factor Model



What to do for longer maturities?

- Accept the model predictions for maturities where there are no futures prices?
- Maybe a few years only?
- Assume flat futures prices?
- Assume futures prices increase at a fixed rate (inflation?)?
- Since there is more uncertainty in this area, what discount rate to use (risk free rate?)?

Recent work on Commodities

- Two recent papers with Anders Trolle
- “Unspanned stochastic volatility and the pricing of commodity derivatives” (2009)
- “Pricing expropriation risk in natural resource contracts – A real options approach” (2010)

Main Issues

- Volatility in commodity markets is stochastic
- Extent to which volatility is spanned by futures prices?
- Are commodity options redundant securities?
- Critical for pricing, hedging and risk management of commodity options and real options
- We analyze these issues in the crude-oil market and develop a new model for pricing commodity derivatives in the presence of unspanned stochastic volatility

NYMEX crude-oil futures and option data

- Largest and most liquid commodity derivatives market in the world
- Largest range of maturities and strike prices, which vary significantly during the sample period
- Daily data from Jan 2, 1990 to May 18, 2006
- We choose the 12 most liquid contracts for the analysis M1, M2, M3, M4, M5, M6 (first 6 monthly contracts) Q1, Q2 (next two quarterly contracts expiring in Mar, Jun, Sep or Dec) Y1, Y2, Y3, Y4 (next four yearly contracts expiring in Dec)

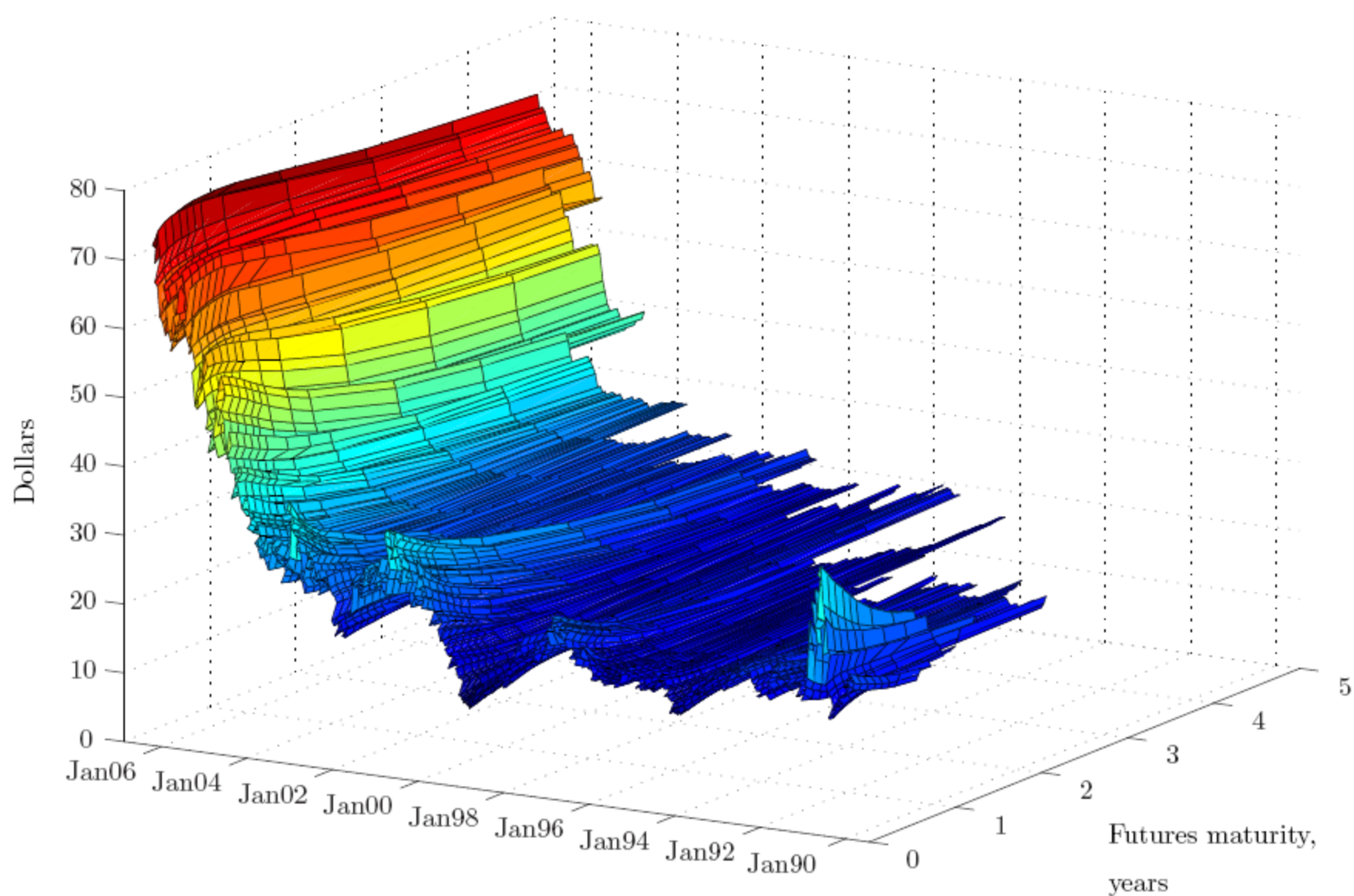


Figure 1: Prices of futures contracts

Prices of M1, M2, M3, M4, M5, M6, Q1, Q2, Y1, Y2, Y3 and Y4 futures contracts. Along the time-dimension there are 855 weekly observations from January 3, 1990 to May 17, 2006.

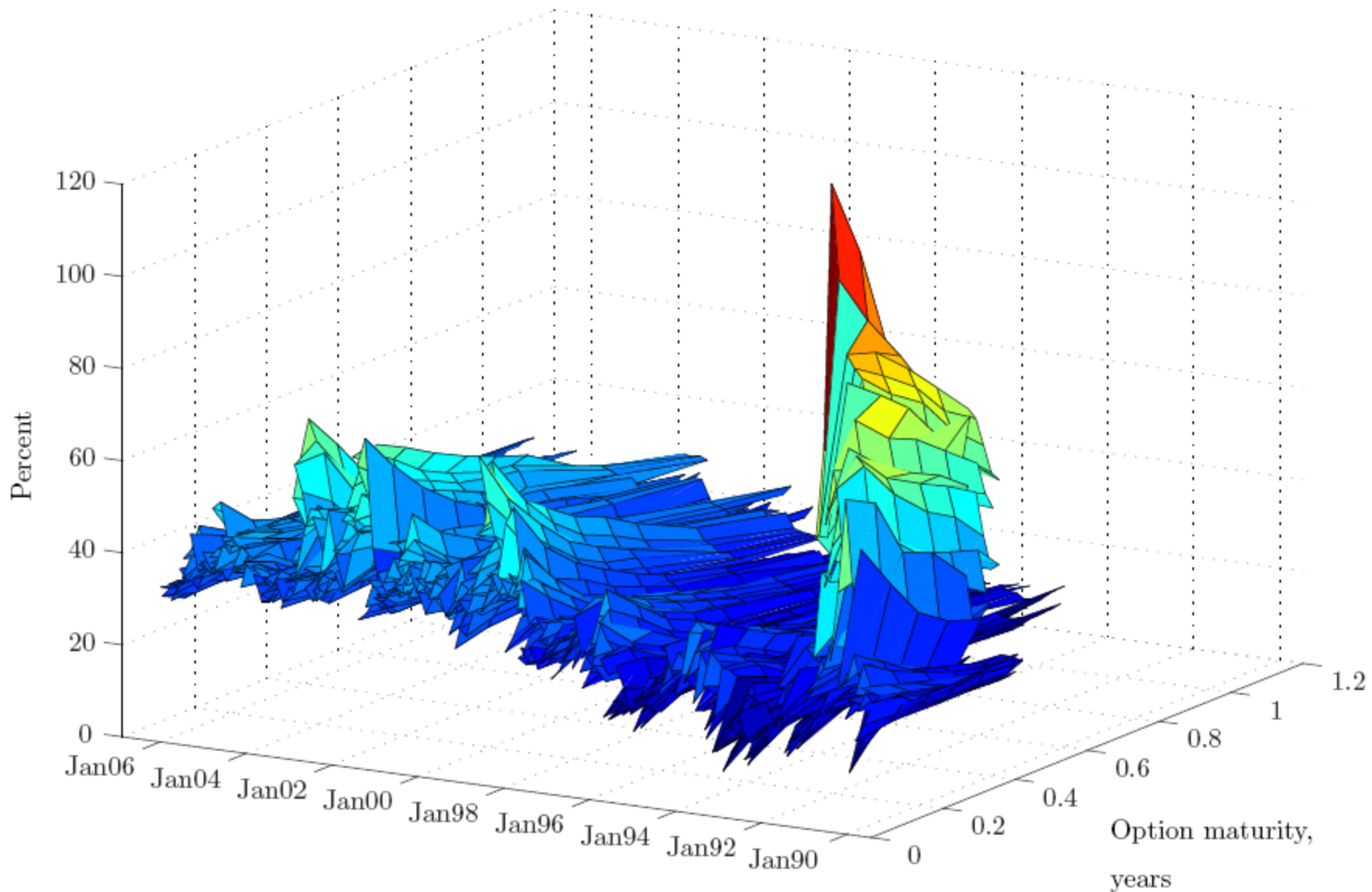


Figure 2: Implied log-normal volatility of futures options

Implied log-normal volatility of options on the M1, M2, M3, M4, M5, M6, Q1 and Q2 futures contracts. Implied volatilities are computed from option prices by inverting the Barone-Adesi and Whaley (1987) formula. Along

Evidence of unspanned stochastic volatility

- If we regress changes in volatility on the returns of futures contracts (or its PC), the R^2 will indicate the extent to which volatility is spanned
- But volatility is not directly observable
- Return on Straddles: Call + Put with the same strike (closest to ATM); low “deltas” and high “vegas”.
Model independent
- Changes in log normal implied volatilities: average expected volatility over the life of the option

Procedure

- We factor analyze the covariance matrix of the futures returns and retain the first three principal components (PCs)
- Regress straddle returns (changes in implied volatilities) on PCs and PCs squared

$$r_t^{strad,i} = \beta_0^i + \beta_1^i PC_t^{fut,1} + \beta_2^i PC_t^{fut,2} + \beta_3^i PC_t^{fut,3} + \beta_4^i \left(PC_t^{fut,1} \right)^2 + \beta_5^i \left(PC_t^{fut,2} \right)^2 + \beta_6^i \left(PC_t^{fut,3} \right)^2 + \epsilon_t^i$$

- We find that the R2 are typically very low, especially for the implied volatility regressions (between 0 and 21%)

Procedure

- Thus, factors that explain futures returns cannot explain changes in volatility
- We then factor analyze the covariance matrix of the residuals from these regressions. If there is unspanned stochastic volatility in the data we should see large common variation in the residuals
- We find that typically the first two PC explain over 80% of the variation in the residuals

Model for pricing commodity derivatives (SV1)

- Model under the risk neutral measure
- Dynamics of the spot price

$$\frac{dS(t)}{S(t)} = \delta(t)dt + \sigma_S \sqrt{v(t)} dW_1^Q(t)$$

- Dynamics of the instantaneous forward cost of carry

$$dy(t, T) = \mu_y(t, T)dt + \sigma_y(t, T) \sqrt{v(t)} dW_2^Q(t)$$

- Dynamics of volatility

$$dv(t) = \kappa(\theta - v(t))dt + \sigma_v \sqrt{v(t)} dW_3^Q(t)$$

- The forward cost of carry is given by the forward interest rate minus the forward convenience yield

Two Volatility Factors (SV2)

$$\frac{dS(t)}{S(t)} = \delta(t)dt + \sigma_{S1}\sqrt{v_1(t)}dW_1^Q(t) + \sigma_{S2}\sqrt{v_2(t)}dW_2^Q(t),$$

$$dy(t, T) = \mu_y(t, T)dt + \sigma_{y1}(t, T)\sqrt{v_1(t)}dW_3^Q(t) \\ + \sigma_{y2}(t, T)\sqrt{v_2(t)}dW_4^Q(t),$$

$$dv_1(t) = (\eta_1 - \kappa_1 v_1(t) - \kappa_{12} v_2(t))dt + \sigma_{v1}\sqrt{v_1(t)}dW_5^Q(t),$$

$$dv_2(t) = (\eta_2 - \kappa_{21} v_1(t) - \kappa_2 v_2(t))dt + \sigma_{v2}\sqrt{v_2(t)}dW_6^Q(t),$$

Pricing expropriation risk in natural resource contracts

- Conference on “The Natural Resources Trap, Private Investment without Public Commitment ” (Kennedy School, Harvard)
- There are many dimensions to the study of expropriation risk in natural resource investments: political, environmental, sociological, economic
- In our approach we abstract from many of these issues and we concentrate on some of the important economic trade-offs that arise from a government having an “option” to expropriate the resource

Expropriation Option

- We value a natural resource project, in particular an oil field, exposed to expropriation risk
- We view the government as holding an option to expropriate the oil field

Expropriation Trade-off

- Government faces the following trade-off in expropriation (think about Argentina vs YPF)
 - Benefit:
 - receives all profits rather than a fraction through taxes
 - Costs:
 - Private firm may produce oil more efficiently
 - Government may have to pay compensation to the firm
 - Government faces “reputational” costs

Assumptions

- We abstract from the various operational options that are typically embedded in natural resource projects and concentrate on the expropriation option
- Spot prices, futures prices and volatilities are described by the one volatility factor model (SV1)

Solution Procedure

- Expropriation option can be exercised at any time during the life of the option: American-style (LSM)
- At every point in time the government must compare the value of immediate exercise with the conditional expected value (under the risk neutral measure) of continuation
- Outcome is the optimal exercise time for each simulated path which can be used to value the expropriation option
- We can also estimate the value of the oil field to the government and the firm both in the presence and absence of expropriation risk

Main Results

- For a given contractual arrangement the value of the expropriation option increases with
 - The spot price
 - The slope of the futures curve (contango, backwardation)
 - The volatility of the spot (futures) price
- For a given set of state variables the value of the expropriation option decreases with the
 - Tax rate
 - Various expropriation costs
- The increase in the field's value to the government due to expropriation risk is always smaller than the decrease in the field value to the firms, since there are “deadweight losses” associated with expropriation (production inefficiency and “reputational costs”)

R&D Investments

Focus has been the pharmaceutical industry
Framework, however, applies just as well to
other research-intensive industries

R&D Investments

- The pharmaceutical industry has become a research-oriented sector that makes a major contribution to health care.
- The success of the industry in generating a stream of new drugs with important therapeutic benefits has created an intense public policy debate over issues such as
 - the financing of the cost of research
 - the prices charged for its products
 - the socially optimal degree of patent protection

- There is a trade-off between promoting innovative efforts and securing competitive market outcomes.
- The expected monopoly profits from drug sales during the life of the patent compensate the innovator for its risky investment.
- The onset of competition after the expiration of the patent limits the deadweight losses to society that arise from monopoly pricing under the patent.
- Regulation has had important effects on the cost of innovation in the pharmaceutical industry.

Analysis of R&D projects is a very difficult investment problem

- Takes a long time to complete (10 to 12 years)
- Uncertainty about costs of development and time to completion (\$4 billion?)
- High probability of failure (for technical or economic reasons) (80%)
- Drug requires approval by the FDA
- Uncertainty about level and duration of future cash flows
- Abandonment option is very valuable
- Yearly US expenditures in prescription drugs :\$308 billion (2010)

Research Spending Per New Drug

Company	Ticker	Number of drugs approved	R&D Spending Per Drug (\$Mil)	Total R&D Spending 1997-2011 (\$Mil)
AstraZeneca	AZN	5	11,790.93	58,955
GlaxoSmithKline	GSK	10	8,170.81	81,708
Sanofi	SNY	8	7,909.26	63,274
Roche Holding AG	RHHBY	11	7,803.77	85,841
Pfizer Inc.	PFE	14	7,727.03	108,178
Johnson & Johnson	JNJ	15	5,885.65	88,285
Eli Lilly & Co. Abbott Laboratories	LLY ABT	11 8	4,577.04 4,496.21	50,347 35,970
Merck & Co Inc	MRK	16	4,209.99	67,360
Bristol-Myers Squibb Co.	BMJ	11	4,152.26	45,675
Novartis AG	NVS	21	3,983.13	83,646
Amgen Inc.	AMGN	9	3,692.14	33,229

Sources: InnoThink Center For Research In Biomedical Innovation; Thomson Reuters Fundamentals via FactSet Research Systems

Pfizer ‘Youth Pill’ Ate Up \$71 Million Before It Flopped (WSJ: May 2, 2002)

- The experimental drug aimed to reverse the physical decline that comes with aging.
- Nearly a decade of research.
- Patients taking the frailty drug had gained some muscle mass – but less than 3% more than the placebo group – which also experienced muscle increase.
- Drug appeared ineffective.

Medivation, Pfizer end work on Alzheimer's Drug (WSJ: January 18, 2012)

- In 2008 Pfizer agreed to pay \$225 million upfront – and up to \$500 million if successful – for development rights to Dimebon.
- Some 5.4 million in the US and 18 million worldwide are estimated to have Alzheimer's
- Analysts say that effective treatment could reach \$25 billion per year

Success story: Lipitor from Pfizer

- Most prescribed name-brand in the US with 3.5 million people taking it every day
- Enter the market in 1997 and loss patent protection at the end of Nov. 2011 with total sales of \$81 billion
- But (WSJ May 2, 2012), Pfizer's first-quarter profit declined 19% as sales of its top product, Lipitor, tumbled 71% in the U.S. amid competition from generic copies.

R&D Valuation

1. Evaluation of Research and Development Investments (with M. Moon, 2000)
2. Patents and R&D as Real Options (2004)
3. R&D Investments with Competitive Interactions (with K. Miltersen, 2004)
4. A Model of R&D Valuation and the Design of Research Incentives (with J. Hsu, 2008)

Patents and R&D as Real Options

- Methodology for the valuation of a single R&D project that is patent protected
- Or equivalently, for determining the value of a patent to develop a particular product

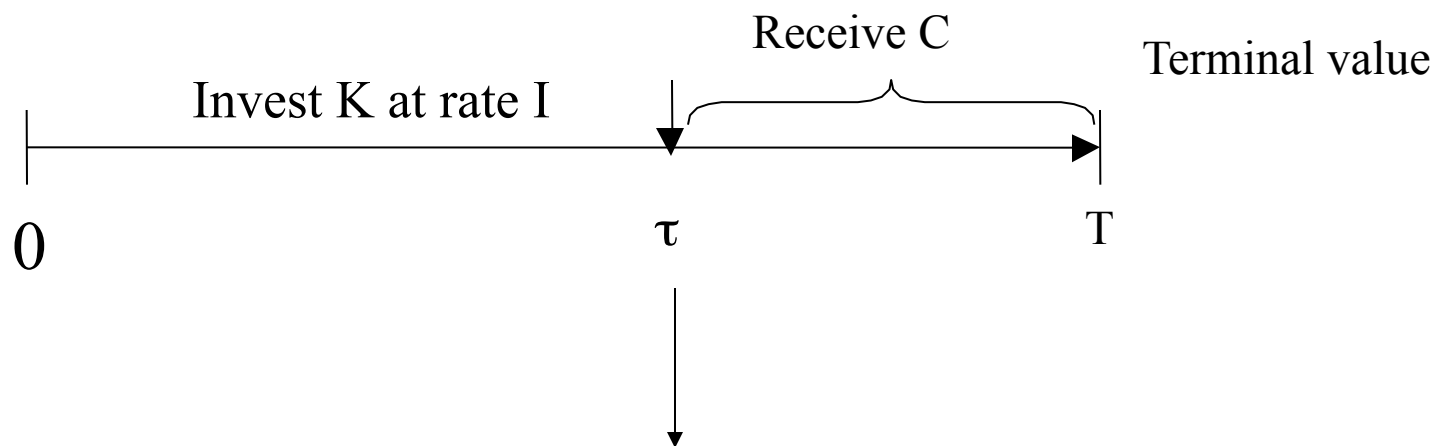
Real Options Approach

- Treat the patent protected R&D project or the patent as a complex option on the variables underlying the value of the project
 - expected costs to completion
 - anticipated cash flows
- Uncertainty is introduced in the analysis by allowing these variables to follow stochastic processes through time
- The risk adjusted process for the cash flows is obtained using the “beta” of traded pharmaceutical companies

Approach

- R&D project takes time to complete
- Maximum rate of investment
- Total cost to completion is random variable
- Probability of failure (catastrophic events)
- Option to abandon the project
- When, and if, project is completed cash flows start to be generated
- Cash flows are uncertain (level and duration)
- Project is patent protected until time T

Patent-protected R&D Project



Investment Cost Uncertainty

Expected cost to completion follows (technical uncertainty):

$$dK = -I dt + \sigma(IK)^{\frac{1}{2}} dz$$

Variance of cost to completion:

$$\text{Var}(\tilde{K}) = \frac{\sigma^2 K^2}{2 - \sigma^2}$$

λ : *Poisson Probability of Failure*

Cash Flow Uncertainty

Cash flow rate follows Geometric Brownian motion which may be correlated with cost process:

$$dC = \alpha C dt + \phi C dw$$

Risk adjusted process used for valuation:

$$dC = (\alpha - \eta) C dt + \phi C dw = \alpha^* C dt + \phi C dw$$

Value of Project once Investment has been Completed: $V(C,t)$

$$\frac{1}{2}\phi^2 C^2 V_{CC} + \alpha^* C V_C + V_t - rV + C = 0$$

Subject to boundary condition at expiration of the patent:

$$V(C, T) = M \cdot C$$

Has solution:

$$V(C, t) = \frac{C}{r - \alpha^*} [1 - \exp(-(r - \alpha^*)(T - t))] + MC \exp(-(r - \alpha^*)(T - t))$$

Stochastic process for the (true) return on the project once investment is completed

$$\frac{dV}{V} = (r + \eta)dt + \phi dw$$

Volatility and risk premium are the same as for cash flows.
Assuming the ICAPM holds the risk premium is:

$$\eta = \beta(r_m - r)$$

Value of the Investment Opportunity: $F(C,K,t)$

$$\text{Max}_I \left[\frac{1}{2} \phi^2 C^2 F_{CC} + \frac{1}{2} \sigma^2 (IK) F_{KK} + \phi \sigma \rho C (IK)^{\frac{1}{2}} F_{CK} + \alpha^* CF_C - \right. \\ \left. IF_K + F_t - (r + \lambda)F - I \right] = 0$$

Subject to boundary condition at completion of investment:

$$F(C,0,\tau) = V(C,\tau)$$

Problem with this is that time of completion is random.

Figure 1
Simulated Paths of Cost to Completion and Quarterly Cash Flow

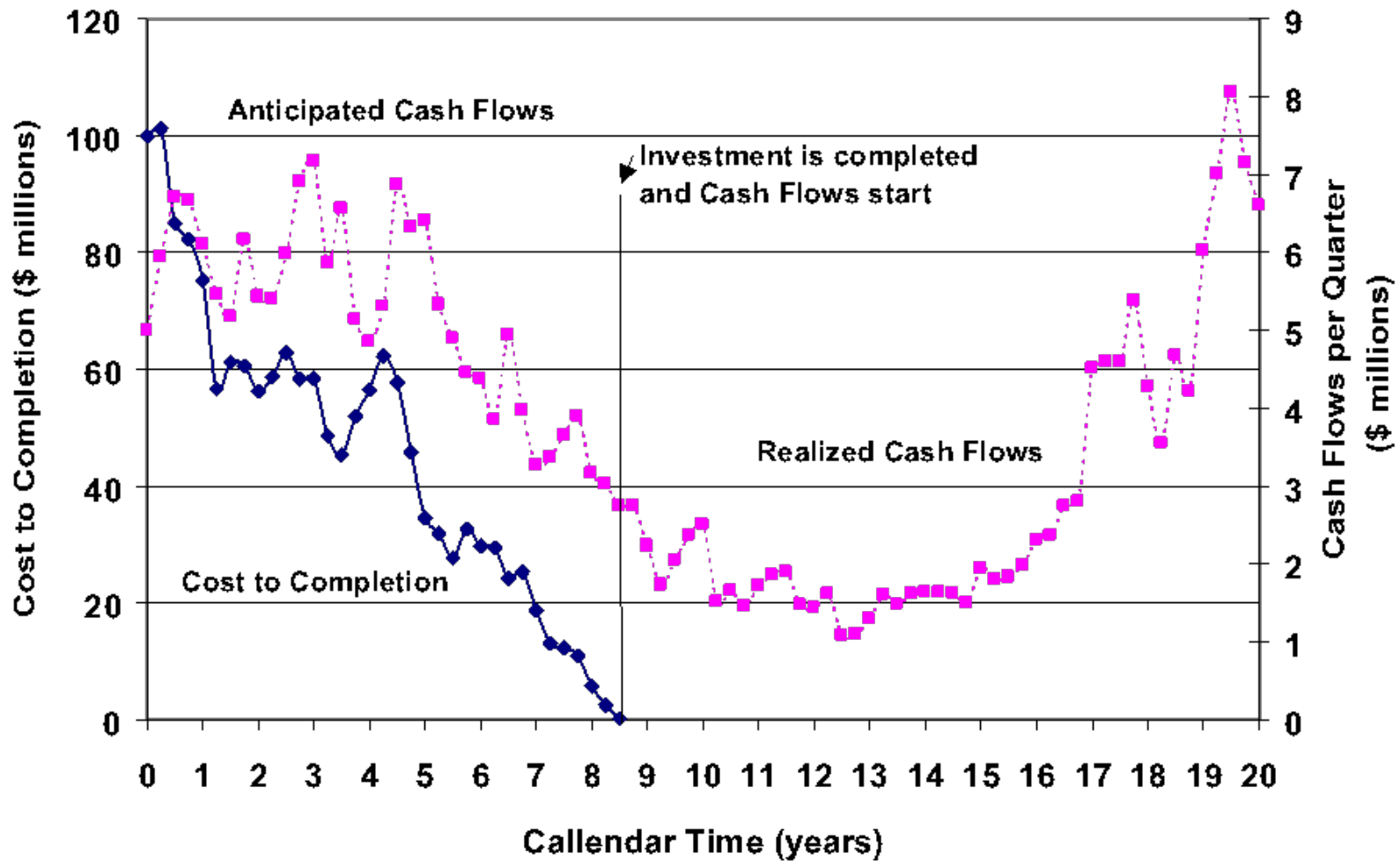
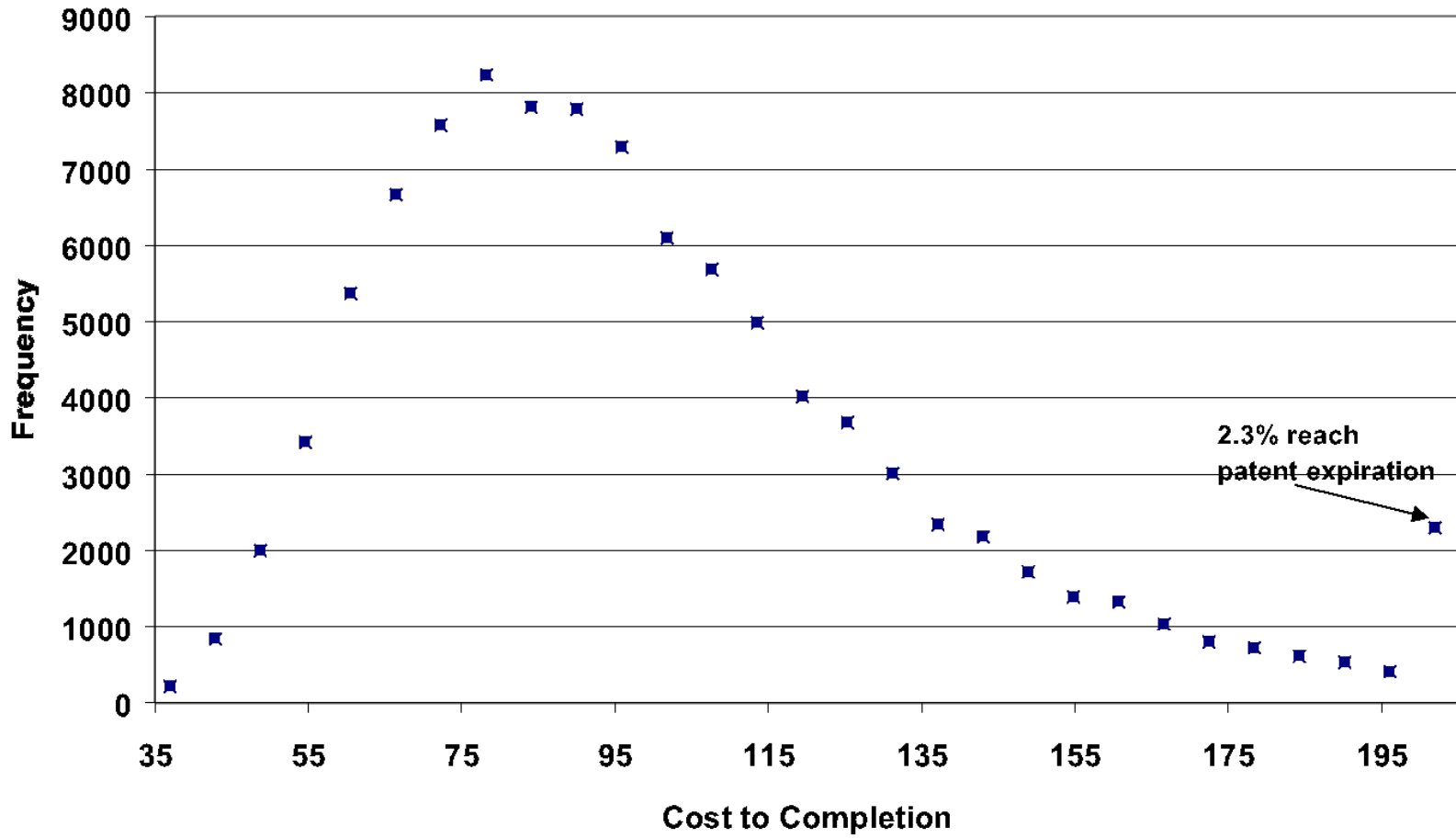
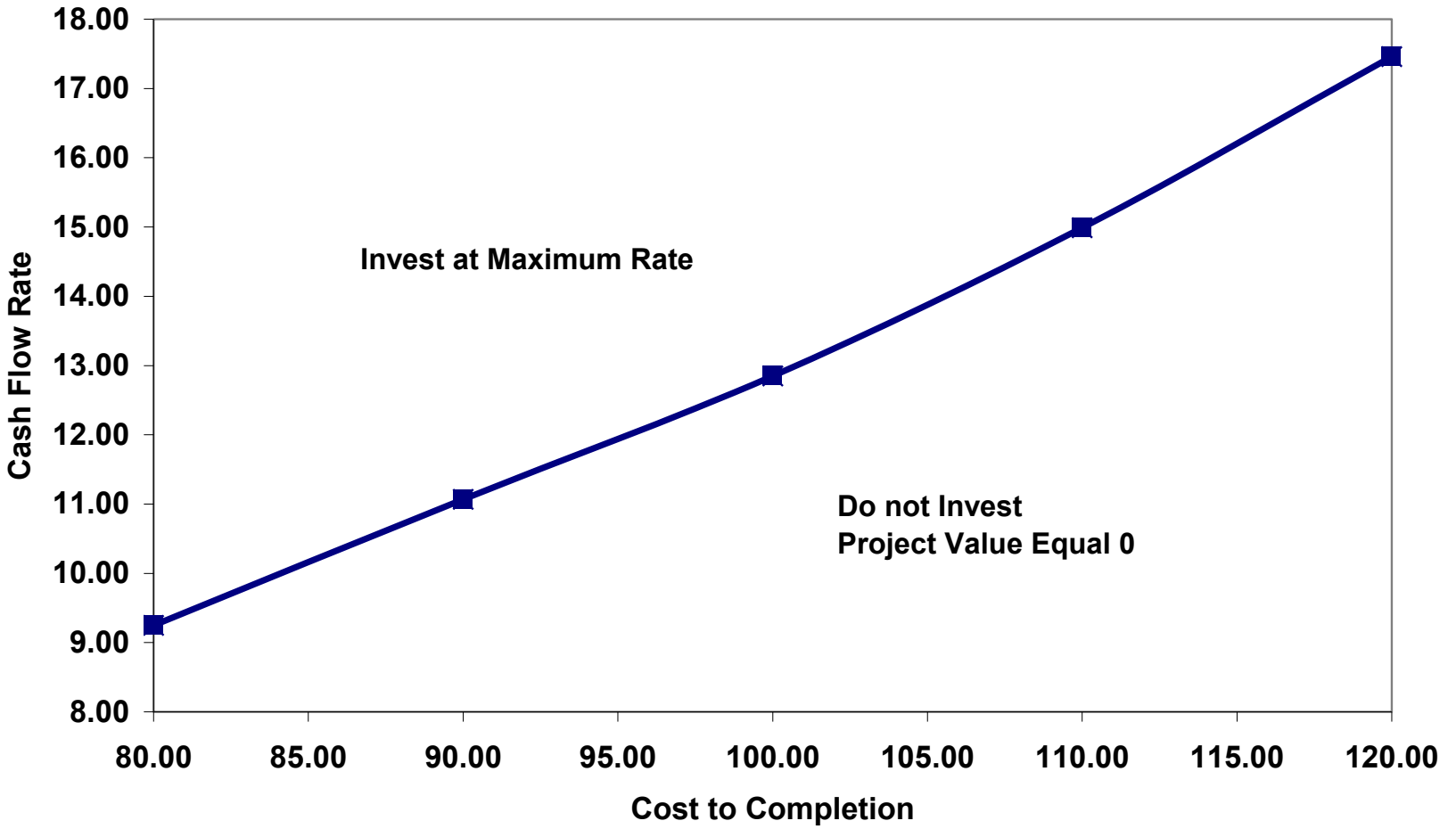


Figure 3
Cost to Completion Distribution



**Figure 4:
Critical Values for Investment**

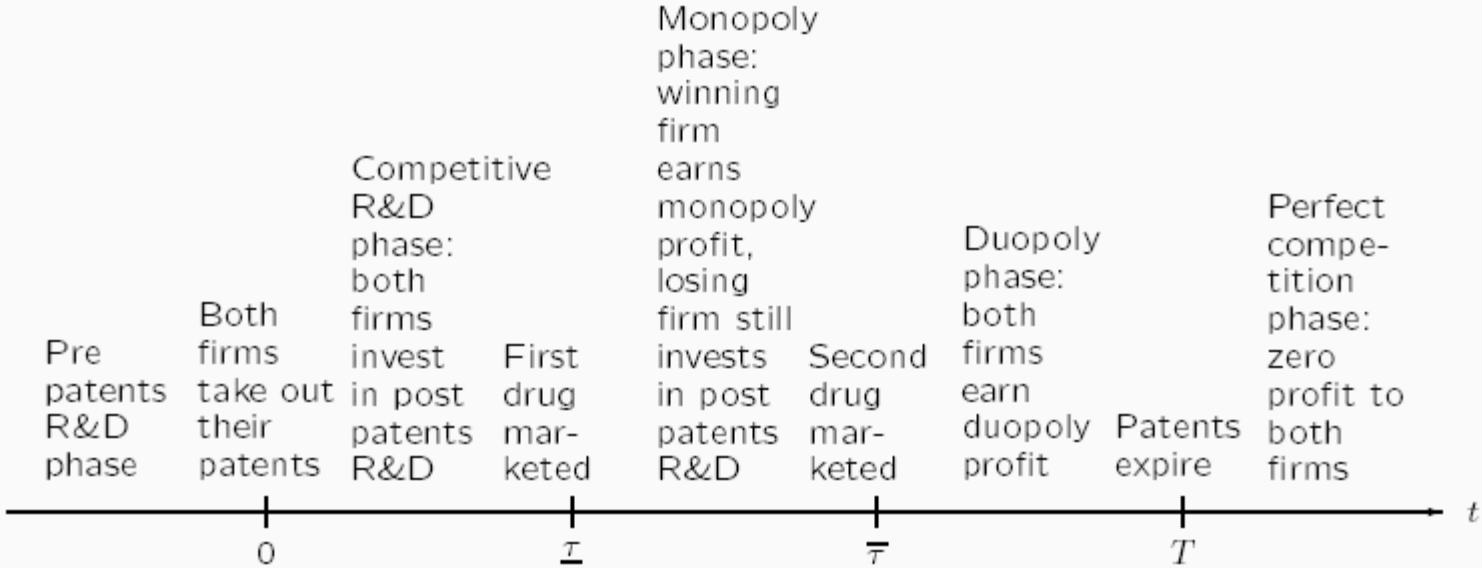


R&D Investments with Competitive Interactions

(joint with K. Miltersen)

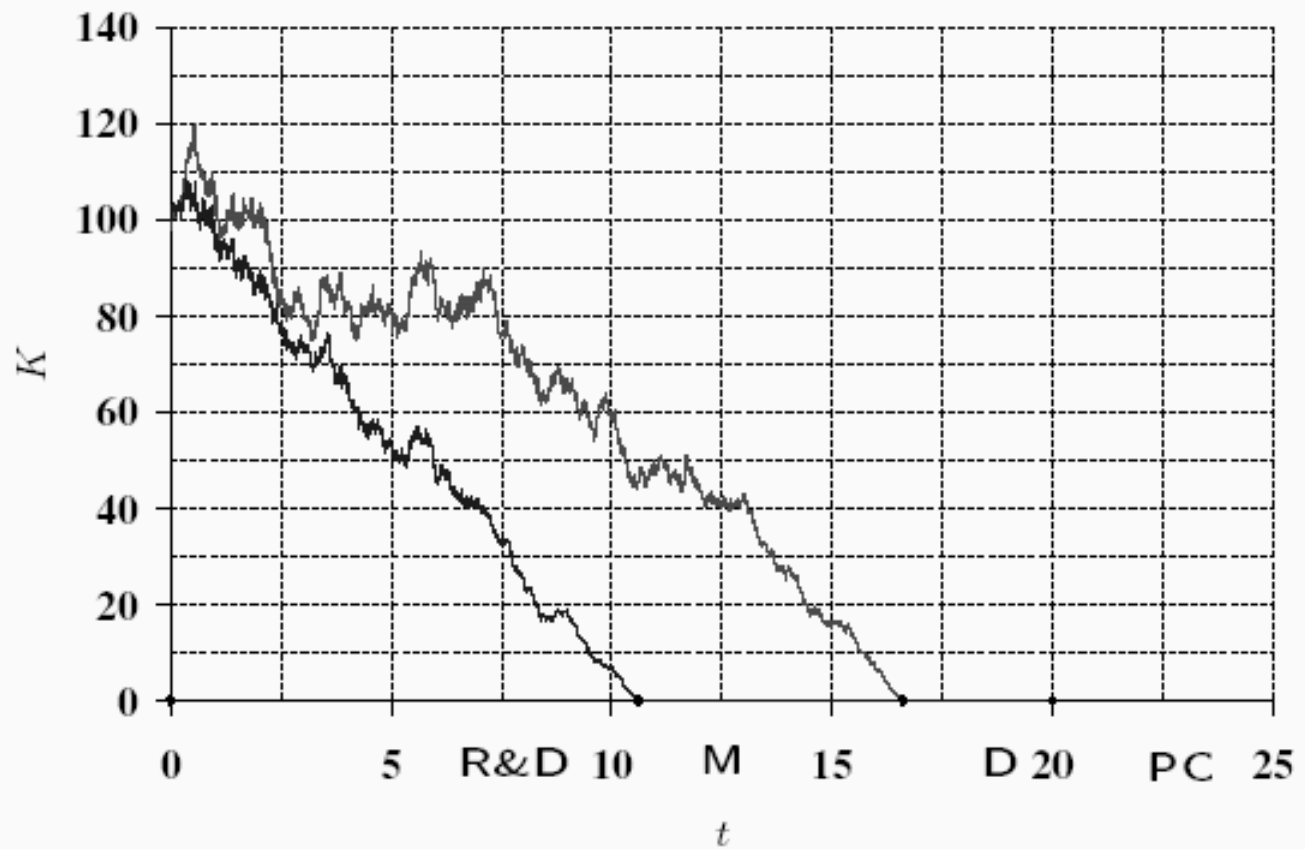
- Concentrates on competitive interactions and the effect it has on valuation and optimal investment strategies
- Real options framework is extended to incorporate game theoretical concepts
- Two firms investing in R&D for different drugs both targeted to cure the same disease
- If both firms successful: Duopoly profits in marketing phase. But it can affect decisions in development phase. Decisions in development phase affects outcome in marketing phase

Time Line of Our Model



Illustrative sample paths

K_1 and K_2



Competition in R&D projects

- Competition brings about
 - Higher production at lower prices
 - Higher probability of success
 - Shorter average development time
- But with higher total development costs and lower values to firms

A Model of R&D Valuation and the Design of Research Incentives (joint with J. Hsu)

- Malaria, Tuberculosis, and African strains of HIV kill more than 5 million people each year
- Almost all of the death occur in the developing world
- Very little private pharmaceutical investment devoted to researching vaccines for these diseases
- A small market problem—people in the developing countries can't afford to pay
- International organizations and private foundations willing to provide funding (WHO, Gates Foundation)

Current Literature on “Encouraging Pharmaceutical Innovation”

- Kremer (2001, 2002) review popular subsidy programs
- Push programs: subsidize the cost of the R&D
 - Research grant
 - Co-payment
- Pull programs: subsidize the revenue of the R&D
 - Purchase commitment
 - Tax incentive
 - Extended patent protection

Current Literature

- No analytical framework for contrasting the different incentive programs

The Contribution

- Develop a real options valuation model for general R&D
- Examine the different incentive programs quantitatively using the valuation framework

What's new in this paper?

- Quality of the R&D output is modeled explicitly
- Revenue is a function of
 - the quality
 - the firm's pricing (and quantity) strategy
 - Market demand
- Firm's price and quantity strategy could depend on
 - Incentive program in place
 - Monopoly power

Analyzing Incentive Contracts

- Push Contracts:
 - Full discretionary research grant
 - Sponsor co-payment
- Pull Contracts:
 - Extended patent protection
 - Fixed price purchase commitment
 - Variable price purchase commitment

Contract Specifics

- Developer retains right, supplies monopoly quantity
 - Full discretionary research grant
 - Sponsor co-payment
 - Patent extension
- Sponsor can contract the socially optimal quantity to be produced
 - Purchase commitment contracts
- We abstract from agency problem arising from asymmetric information between vaccine developer and sponsor, and from contracting issues

We seek to answer five critical questions

- What is the required level of monetary incentive to induce the firm to undertake the vaccine R&D?
- What are the expected price, quantity supplied and efficacy of the developed vaccine?
- What is the probability that a viable vaccine will be developed?
- What is the consumer surplus generated?
- What is the expected cost per individual successfully vaccinated?